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TECHNICAL NOTE R-51

A NUMERICAL METHOD FOR COMPUTING TRANSMISSION AND REFLECTION COEFFICIENTS OF AN INHOMOGENEOUS PLASMA LAYER

Prepared By

J. M. Scarborough

June, 1963

BROWN ENGINEERING COMPANY INC.
HUNTSVILLE, ALABAMA

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A NUMERICAL METHOD FOR COMPUTING TRANSMISSION AND REFLECTION COEFFICIENTS OF AN INHOMOGENEOUS PLASMA LAYER

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ABSTRACT

A numerical method of computing reflection and transmission coefficients for inhomogeneous plasma layers when the gradient of inhomogeneity is normal to the surface of the layer is presented. The method is applied to a specific problem of telemetry from a body re-entering the earth's atmosphere and the results are discussed.

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Harry C. Crews, Jr.
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LIST OF SYMBOLS

A	Absorption coefficient
$\vec{B}(t)$	Time dependent magnetic induction vector
$\vec{D}(t)$	Time dependent electric displacement vector
$\vec{E}(t)$	Time dependent electric intensity vector
$\vec{E}$	Time independent electric field intensity
E	Magnitude of electric field intensity within plasma
E^I	Magnitude of incident electric field intensity
E^R	Magnitude of reflected electric field intensity
E^T	Magnitude of transmitted electric field intensity
E_o^I	Amplitude of incident electric field intensity
E_o^R	Amplitude of reflected electric field intensity
E_i	Imaginary part of E(o)
E_r	Real part of E(o)
e	Base of Napierian logarithms
e	Electron charge
$\vec{H}(t)$	Time dependent magnetic intensity vector
$\vec{H}$	Time independent magnetic field intensity
H	Magnitude of magnetic field intensity within plasma
H^T	Magnitude of transmitted magnetic field intensity
H_o^I	Amplitude of incident magnetic field intensity
H_o^R	Amplitude of reflected magnetic field intensity

H_i	Imaginary part of H(o)
H_r	Real part of H(o)
i	$\sqrt{-1}$
$\vec{J}$	Time dependent true current density
k	Wave number of incident wave
k_o	Wave number of transmitted wave in free space
m	Electron mass
$\vec{n}$	Unit vector in +z direction
n	Electron number density in the plasma
R	Reflection coefficient
R_n	Nose radius of re-entry body
T	Transmission coefficient
t	Time
$\vec{v}$	Velocity of electron
z	Cartesian coordinate normal to surface of plasma layer
z_o	Point on z axis in free space region

<u>Greek Symbols</u>

ϵ_m	Permittivity of dielectric adjoining plasma layer
ϵ_o	Permittivity of free space
ϵ	Effective permittivity of plasma
μ	Effective permeability of plasma
μ_o	Permeability of free space

Greek Symbols (cont.)

ν	Frequency of collisions of electrons with neutral particles
σ	Effective conductivity of plasma
ω	angular frequency of incident wave
ω_p	angular plasma frequency

INTRODUCTION

In attempting telemetry of data from test bodies re-entering the earth's atmosphere, the problem of penetrating with an electromagnetic wave the sheath of ionized gases (plasma) which envelopes these bodies at hypersonic velocities is encountered. A knowledge of reflection and transmission coefficients for the sheath is of value in attempts at solution of this problem.

This paper presents a method for computing reflection and transmission coefficients for a plane-parallel inhomogeneous isotropic layer of plasma when the inhomogeneity is a function only of distance along a normal to the surface of the layer. A plane wave at normal incidence is assumed.

It is recognized that as a model for the plasma sheath described above, the idealization treated is deficient in several important respects. It neglects the curvature of the layer as well as the fact that the incident wave is not plane. The effects of induction fields near the antenna and the problem of antenna breakdown are similarly ignored.

Despite these objections, it is believed that solutions of the simplified problem may be taken as order of magnitude estimates of the transparencies of sheaths having the prescribed distributions of electrical properties. Moreover, the results should be of interest for comparison with those of more comprehensive calculations.

This and related problems have been treated by several authors, but most of these neglect the effect of spatial variation of collision frequency which Kritz[7] has shown to be significant (in certain cases) in determining the reflection and transmission characteristics of the plasma.

Exact analytic solutions in closed form are possible only for the simpler distributions of permittivity and conductivity, and more complicated distributions have been treated using various approximation methods[1, 2, 6, 8]. Existing solutions of these types for a few simple plasma geometries are discussed and tabulated by Graf and Bachynski[3]. These solutions, however, have rather limited applicability and may prove cumbersome in calculations of reflection and transmission coefficients. A direct and expedient method for computing these coefficients for a wide variety of distributions of both permittivity and conductivity which takes advantage of available automatic computation facilities is needed.

These requirements are probably best met by a method involving the direct numerical integration of Maxwell's equations within the plasma. This approach is not new, and several workers in the field have discussed the relative merits of different forms for the equations and of various integration processes. Kritz[7] presents a method, and Zivanovic[10] uses a numerical technique to compute a set of matrix elements descriptive of the plasma layer after the manner of four terminal networks and transmission lines in network theory. Klein[6] and Budden[1] treat the problem of joining approximate analytic solutions with numerical solutions at boundaries separating the regions of validity of the two. Each of these approaches offers certain advantages.

The computer program used provides flexibility in the choice of distributions of both conductivity and permittivity and the modified Runge-Kutta integration process employed allows for control of accuracy in the solution. The program is described in greater detail in a report by Kavanaugh and Scarborough[5].

ANALYSIS

Maxwell's equations for a stationary medium containing no free charges are:

$$
\begin{aligned}
\nabla \cdot \vec{D}(t) &= 0 \\
\nabla \cdot \vec{B}(t) &= 0 \\
\nabla \times \vec{E}(t) &= -\frac{\partial \vec{B}(t)}{\partial t} \\
\nabla \times \vec{H}(t) &= \vec{J} + \frac{\partial \vec{D}(t)}{\partial t} \;.
\end{aligned}
\tag{1}
$$

For an inhomogeneous isotropic plasma, the constitutive equations may be written

$$
\begin{aligned}
\vec{B}(t) &= \mu \vec{H}(t) \\
\vec{D}(t) &= \epsilon \, \vec{E}(t) \\
\vec{J} &= \sigma \vec{E}(t)
\end{aligned}
\tag{2}
$$

with μ , ϵ, and σ scalar functions of position.

In a plasma it is observed that the effective permeability is very nearly equal to the free-space value μ_o. It has been shown in several works [5, 6, 8,] that, if only those fields whose time dependence may be expressed as $e^{-i\omega t}$ are considered, the conductivity effective in the plasma is given by the relation

$$
\sigma = \frac{\nu n e_e^2}{m(\nu^2 + \omega^2)} \;. \tag{3}
$$

This expression is arrived at by considering the motion of an electron under the influence of an electromagnetic wave and subject to damping by collisions with heavier particles. The $\vec{v} \times \vec{B}(t)$ term in the Lorentz force is found to be negligible.

The same analysis leads to the following expression for the permittivity effective in the plasma:

$$\epsilon = \epsilon_o \left[1 - \frac{ne^2}{m\epsilon_o(\nu^2 + \omega^2)} \right] . \tag{4}$$

In terms of the plasma frequency $\omega_p^2 \equiv \frac{ne^2}{m\epsilon_o}$ the expressions (3) and (4) are

$$\sigma = \epsilon_o \frac{\nu\omega_p^2}{(\nu^2 + \omega^2)} \tag{5}$$

and

$$\epsilon = \epsilon_o \left[1 - \frac{\omega_p^2}{\nu^2 + \omega^2} \right] . \tag{6}$$

Writing $\vec{E}(t) = \vec{E}\, e^{-i\omega t}$ and $\vec{H}(t) = \vec{H}\, e^{-i\omega t}$ and using the constitutive equations (2), Maxwell's equations (1) become

$$\begin{aligned} \nabla \cdot \epsilon \vec{E} &= 0 \\ \nabla \cdot \vec{H} &= 0 \\ \nabla \times \vec{E} &= i\omega\mu_o \vec{H} \\ \nabla \times \vec{H} &= (\sigma - i\omega\epsilon) \vec{E} . \end{aligned} \tag{7}$$

Confining attention to the particular geometry of interest, if the spatial variations of the field quantities and ε and σ depend on a single coordinate z normal to the surface of the layer, the harmonic fields are transverse and the equations (7) reduce to

$$
\begin{aligned}
\vec{n} \cdot \frac{\partial \varepsilon \vec{E}}{\partial z} &= 0 \\
\vec{n} \cdot \frac{\partial \vec{H}}{\partial z} &= 0 \\
\vec{n} \times \frac{\partial \vec{E}}{\partial z} &= i \omega \mu_o \vec{H} \\
\vec{n} \times \frac{\partial \vec{H}}{\partial z} &= (\sigma - i \omega \varepsilon) \vec{E}
\end{aligned}
\tag{8}
$$

in which $\vec{n}$ is a unit vector along the z axis. Expanding the first of these yields the relation

$$\vec{n} \cdot \vec{E} \frac{\partial \varepsilon}{\partial z} + \varepsilon \vec{n} \cdot \frac{\partial \vec{E}}{\partial z} = 0 . \tag{9}$$

The first term of (9) represents a coupling between the electric field vector and the gradient of the inhomogeneity in permittivity which is here zero since $\vec{n} \cdot \vec{E} = 0$ for a transverse field and normal incidence.

Hence, the first of equations (8) becomes

$$\vec{n} \cdot \frac{\partial \vec{E}}{\partial z} = 0 .$$

A further simplification of the equations is possible, since only the magnitudes of the field need be considered, their directions in space being constant. The third and fourth of equations (8) then reduce to

$$\frac{dE}{dz} = i\omega\mu_o H$$
$$\frac{dH}{dz} = i(\epsilon\omega + i\sigma) E, \tag{10}$$

the first two equations yielding no additional information concerning the fields.

Equations (10) have been solved exactly for only a few simple distributions, $\epsilon(z)$ and $\sigma(z)$ and the general case must be treated by various approximation methods or by numerical techniques. Since in the present instance, numerical values for reflection and transmission coefficients are of greater interest than a general solution to (10) and facilities for high-speed automatic computation are available, the latter alternative is favored.

Following a procedure suggested by the work of Kritz[7] and Budden[1], a transmitted wave E^T of a particular amplitude (unity) and phase $(k_o z - k_o z_o)$, is assumed in the free space region immediately "outside" $(z \geq z_o)$ the plasma layer. (See Figure 1.) With $E^T(z_o) = 1$ and $H^T(z_o) = \sqrt{\frac{\epsilon_o}{\mu_o}}$ as initial values, the four simultaneous equations represented by (10) are integrated numerically over the region $0 \leq z \leq z_o$,

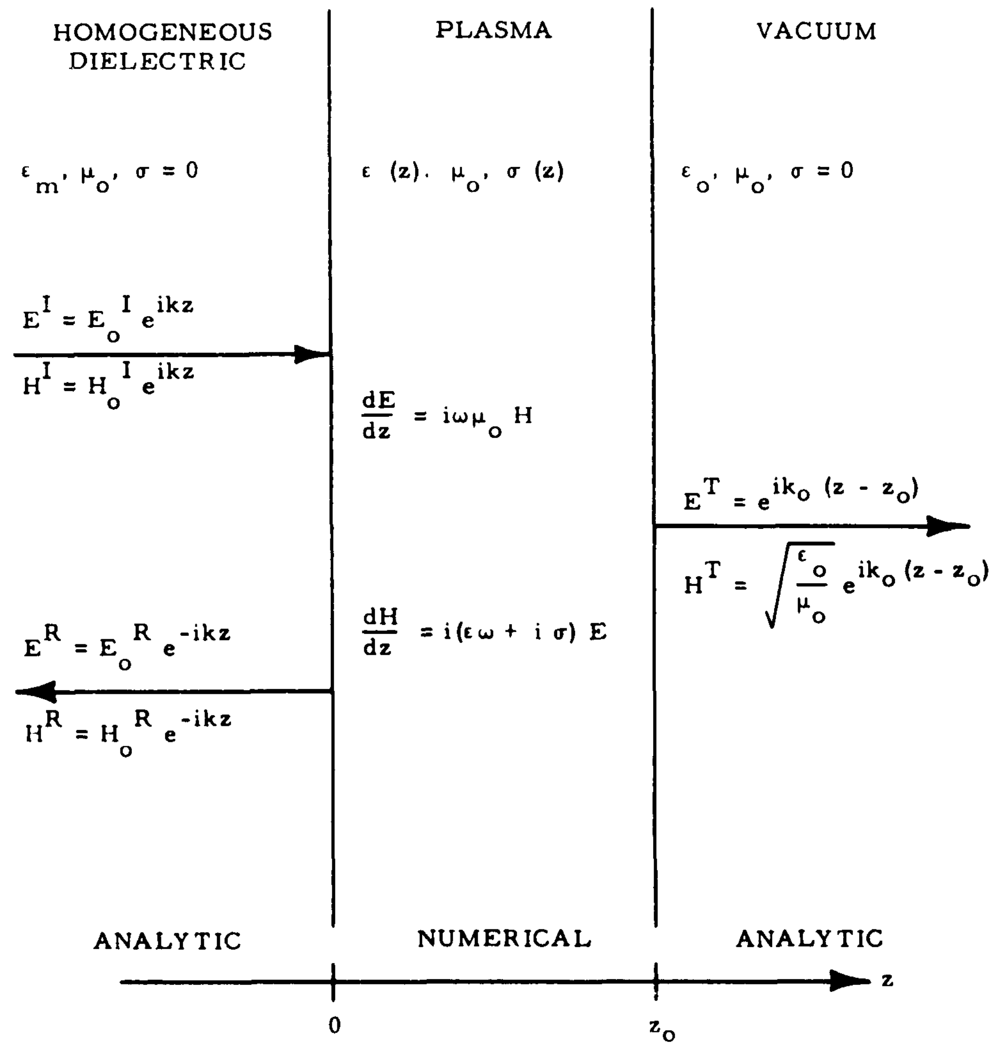

Figure 1. Schematic Representation Showing the Type of Solution Valid in Each Region

the integration proceeding backwards along the z axis. At the boundary $z = 0$, it is required that the E and H fields be continuous, i.e., that

$$\begin{aligned} E^I(0) + E^R(0) &= E(0) \\ H^I(0) + H^R(0) &= H(0) \quad . \end{aligned} \tag{11}$$

But

$$\begin{aligned} E^I(0) &= E_o^I , \\ E^R(0) &= E_o^R , \\ H^I(0) &= \sqrt{\frac{\epsilon_m}{\mu_o}}\, E_o^I , \end{aligned} \tag{12}$$

and

$$H^R(0) = -\sqrt{\frac{\epsilon_m}{\mu_o}}\, R_o^R .$$

Substituting from (12) into (11), the following pair of simultaneous equations is obtained:

$$\begin{aligned} E_o^I + E_o^R &= E(0) \\ E_o^I - E_o^R &= \sqrt{\frac{\mu_o}{\epsilon_m}}\, H(0) \quad . \end{aligned} \tag{13}$$

Solving for E_o^I and E_o^R gives

$$E_o^I = \tfrac{1}{2}\left[E(0) + \sqrt{\frac{\mu_o}{\epsilon_m}}\, H(0)\right] \tag{14}$$

and

$$E_o^R = \tfrac{1}{2}\left[E(0) - \sqrt{\frac{\mu_o}{\epsilon_m}}\, H(0)\right] \quad .$$

The reflection coefficient, defined by

$$R \equiv \frac{|E_o^R|^2}{|E_o^I|^2} \tag{15}$$

becomes, in terms of E_r, E_i, H_r, H_i defined by $E(o) \equiv E_r + i E_i$ and $H(o) \equiv H_r + i H_i$,

$$R = \frac{(\sqrt{\epsilon_m}\, E_r - \sqrt{\mu_o}\, H_r)^2 + (\sqrt{\epsilon_m}\, E_i - \sqrt{\mu_o}\, H_i)^2}{(\sqrt{\epsilon_m}\, E_r + \sqrt{\mu_o}\, H_r)^2 + (\sqrt{\epsilon_m}\, E_i + \sqrt{\mu_o}\, H_i)^2} \; . \tag{16}$$

Similarly, the transmission coefficient is

$$T \equiv \sqrt{\frac{\epsilon_o}{\epsilon_m}}\, \frac{|E_o^T|^2}{|E_o^I|^2} = \frac{4\sqrt{\epsilon_m\, \epsilon_o}}{(\sqrt{\epsilon_m}\, E_r + \sqrt{\mu_o}\, H_r)^2 + (\sqrt{\epsilon_m}\, E_i + \sqrt{\mu_o}\, H_i)^2} \; . \tag{17}$$

Requiring conservation of energy gives for the absorption coefficient

$$A = 1 - (R + T) \; . \tag{18}$$

EXAMPLE

The following study will serve as an example of the type problem to which the method is applicable:

It is required to find transmission and reflection coefficients for the plasma sheath surrounding certain bodies during re-entry at an altitude of 80 km and at speeds of Mach 18 and Mach 30. Various positions on the body surface are to be considered in order to determine the most favorable location for a transmitting slot antenna, and the effect on transmission of varying the dielectric constant of the "window" covering the slot is to be investigated. A transmitting frequency of 240 Mc/s is assumed.

Electrical properties of the plasma were computed from chemical equilibrium flow field values of pressure and enthalpy. Two points on the body surface (at 2 R_n and 4 R_n, as measured along the axis of symmetry, where R_n is the nose radius of the body) were selected as representative, (Figure 2). Profiles of relative permittivity and conductivity along lines normal to the body surface at these points were taken as the distributions of electrical properties within a plane-parallel plasma slab and along a normal to its surface. A more complete description of the above procedure is to be found in the appendix, together with curves representing the distributions prevailing at the various speeds and positions.

The distributions were approximated by the following analytical expressions:

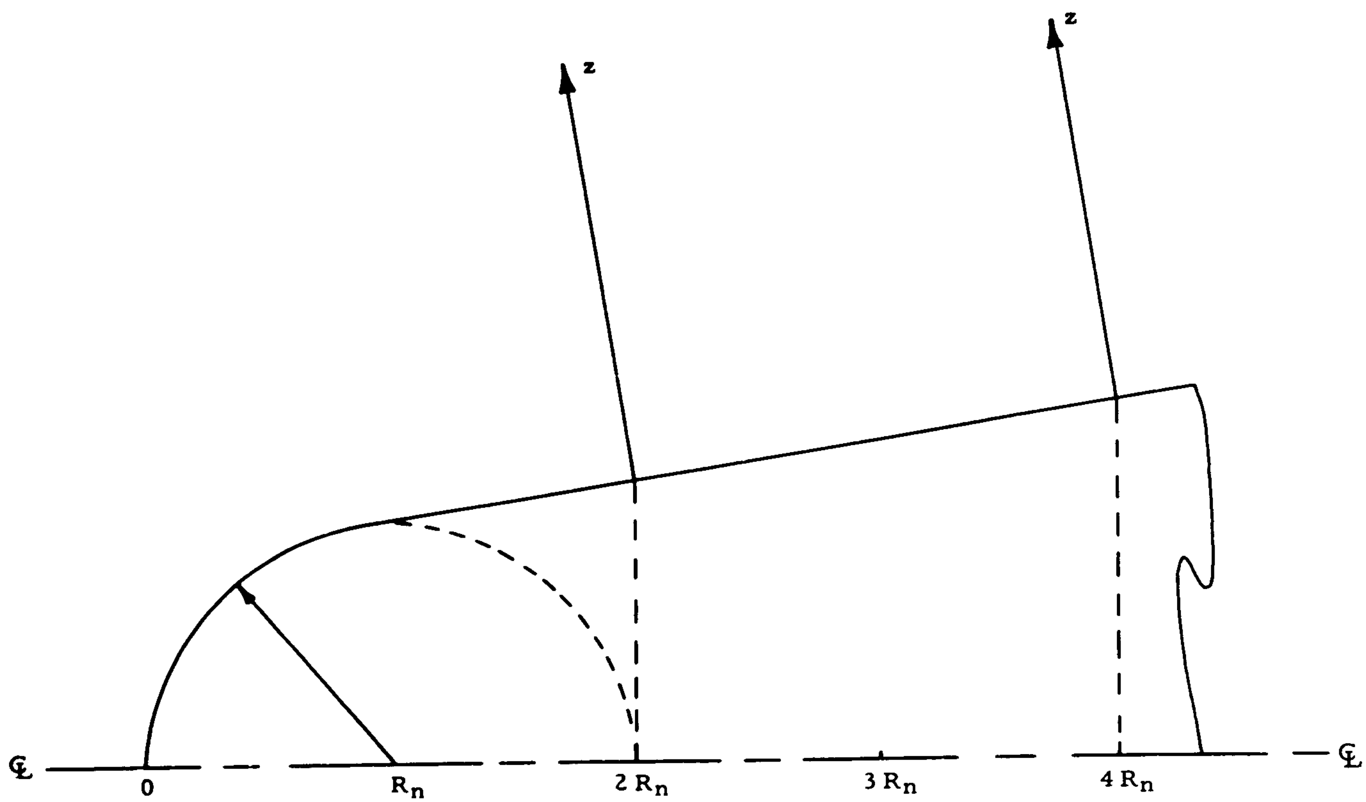

Figure 2

Geometry of Re-entry Body

Mach 18

$2R_n$

$$\frac{\varepsilon}{\varepsilon_o} = \begin{cases} 1 - 0.4\, e^{-108\, z} , & 0 \leq z \leq 0.095 \\ 1 , & z > 0.095 \end{cases}$$

$$\frac{\sigma}{\varepsilon_o \omega} = \begin{cases} 0.04\, e^{-38.4\, z} , & 0 \leq z \leq 0.095 \\ 0 , & z > 0.095 \end{cases}$$

$4R_n$

$$\frac{\varepsilon}{\varepsilon_o} = \begin{cases} 1 - 0.09\, e^{-56.8\, z} , & 0 \leq z \leq 0.095 \\ 1 . & z > 0.095 \end{cases}$$

$$\frac{\sigma}{\varepsilon_o \omega} = \begin{cases} 0.0135\, e^{-28.3\, z} . & 0 \leq z \leq 0.095 \\ 0 . & z > 0.095 \end{cases}$$

Mach 30

$2R_n$

$$\frac{\varepsilon}{\varepsilon_o} = \begin{cases} 2500\, z - 169 , & 0 \leq z \leq 0.0681 \\ 1 , & z > 0.0681 \end{cases}$$

$$\frac{\sigma}{\varepsilon_o \omega} = \begin{cases} -8.7[illegible]\, z + 0.86 , & 0 \leq z \leq 0.0980 \\ 0 , & z > 0.0980 \end{cases}$$

$4R_n$

$$\frac{\varepsilon}{\varepsilon_o} = \begin{cases} 1045\, z - 101 , & 0 \leq 0.0976 \\ 1 , & z > 0.976 \end{cases}$$

$$\frac{\sigma}{\varepsilon_o \omega} = \begin{cases} -5.59\, z + 0.534 , & 0 \leq z \leq 0.0954 \\ 0 , & z > 0.0954 \end{cases}$$

$z_o = 0.1$ m $\qquad \omega = 0.1508 \times 10^{10}$ radian/sec

RESULTS

The reflection and transmission coefficients computed using these distributions and for several values of ε_m are tabulated below:

Mach 30

$\varepsilon_m/\varepsilon_o$	2 Rn R	2 Rn T	4 Rn R	4 Rn T
1	0.998	0.529×10^{-3}	0.998	0.478×10^{-3}
2	0.998	0.742×10^{-3}	0.997	0.669×10^{-3}
3	0.997	0.903×10^{-3}	0.997	0.810×10^{-3}
4	0.997	0.104×10^{-2}	0.997	0.925×10^{-3}
5	0.997	0.115×10^{-2}	0.996	0.102×10^{-2}

Mach 18

$\varepsilon_m/\varepsilon_o$	2 Rn R	2 Rn T	4 Rn R	4 Rn T
1	0.989×10^{-4}	0.995	0.175×10^{-4}	0.998
2	0.290×10^{-1}	0.996	0.293×10^{-1}	0.968
3	0.711×10^{-1}	0.924	0.716×10^{-1}	0.926
4	0.110×10^{0}	0.885	0.111×10^{0}	0.887
5	0.145×10^{0}	0.851	0.146×10^{0}	0.852

Reflection and Transmission Coefficients for the Plasma Sheath Surrounding a Body Re-entering the Atmosphere at Mach 18 and Mach 30, at Two Positions on the Surface and for Various Dielectric Constants of the Antenna Window.

CONCLUSIONS

Several conclusions of a qualitative nature may be drawn from these results. At Mach 18, transmission is almost complete with very little reflection occurring, whereas, at Mach 30, the situation is reversed. It will be noted that in neither case is absorption of energy responsible for appreciable loss in transmission, since the absorption coefficient $A \equiv 1 - (R + T)$ is in every case very small. The effect is due almost entirely to increased reflection.

Also of interest is the effect on transmission of the value of permittivity of the "window". At Mach 18, an increase in this permittivity results in a decrease in transmitted power, whereas the opposite is true at Mach 30. It is suggested that further investigation of this effect is needed to determine whether it is significant. The results further indicate that at Mach 30, absorption is more pronounced at the $4R_n$ position than at $2R_n$, as evidenced by the smaller values for both R and T at this position. The reason for this is not yet apparent, but the difference in wake thickness, together with the fact that ϵ has large negative values at this speed, are probably responsible.

Summarizing, the results indicate that for the assumed transmission frequency, failure to penetrate the Mach 18 sheath may not be attributed to reflection or absorption of the electromagnetic energy in the wave by the plasma through the mechanisms considered here. At Mach 30, however, these effects almost certainly will preclude transmission at this frequency and at any position on the body surface.

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APPENDIX

Values of pressure and enthalpy in the flow field surrounding a particular body were computed assuming chemical equilibrium for speeds of Mach 18 and Mach 30 at an altitude of 80 km.[d]

From these values, the corresponding temperatures and mass density ratios were determined from a Mollier diagram of properties of equilibrium air.[a] Electron densities were then read from a second chart giving electron density as a function of mass density ratio for various temperatures.[b] Values for the collision frequency ν were determined from the formula

$$\nu = (1.10 \times 10^{15}) \times \frac{\text{pressure in atmospheres}}{\text{temperature in }^\circ\text{K}}$$ [c]

Using the values of n and ν thus determined, σ and ϵ were computed using equations (3) and (4). Contours of constant ϵ/ϵ_0 are plotted in Figures 3 and 5, and contours of constant $\sigma/e_0\omega$ are plotted in Figures 4 and 6, and approximate location of the shock wave is also shown.

Plots of ϵ/ϵ_0 and $\sigma/\epsilon_0\omega$ as functions of distance along the normals are given in Figures 7 through 10. The Mach 18 curves are approximated by exponential functions; the Mach 30 curves by linear functions. These analytic approximations appear in the main body of the report.

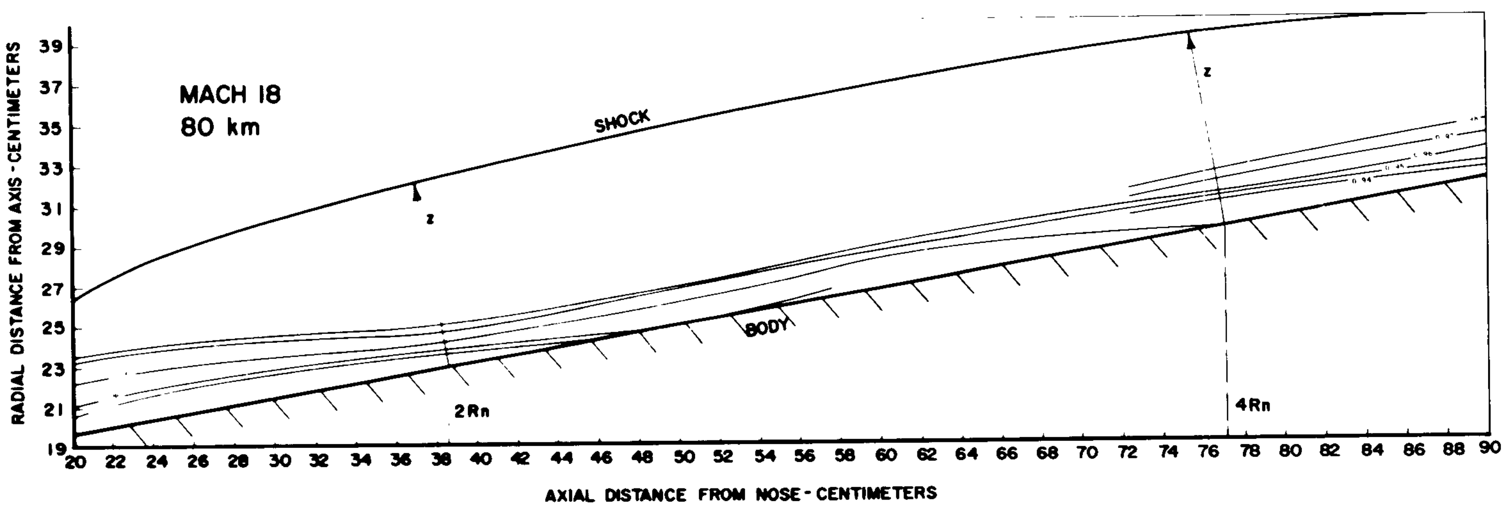

FIGURE 3. CONTOURS OF CONSTANT ϵ/ϵ_0

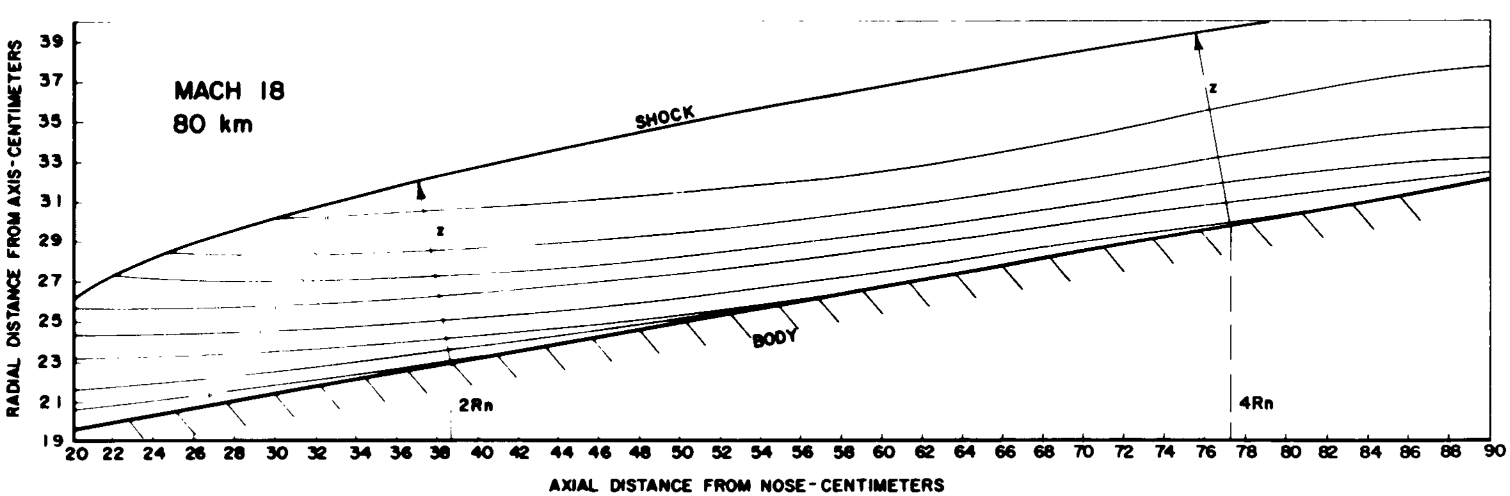

FIGURE 4. CONTOURS OF CONSTANT $\sigma/\epsilon_0\omega$

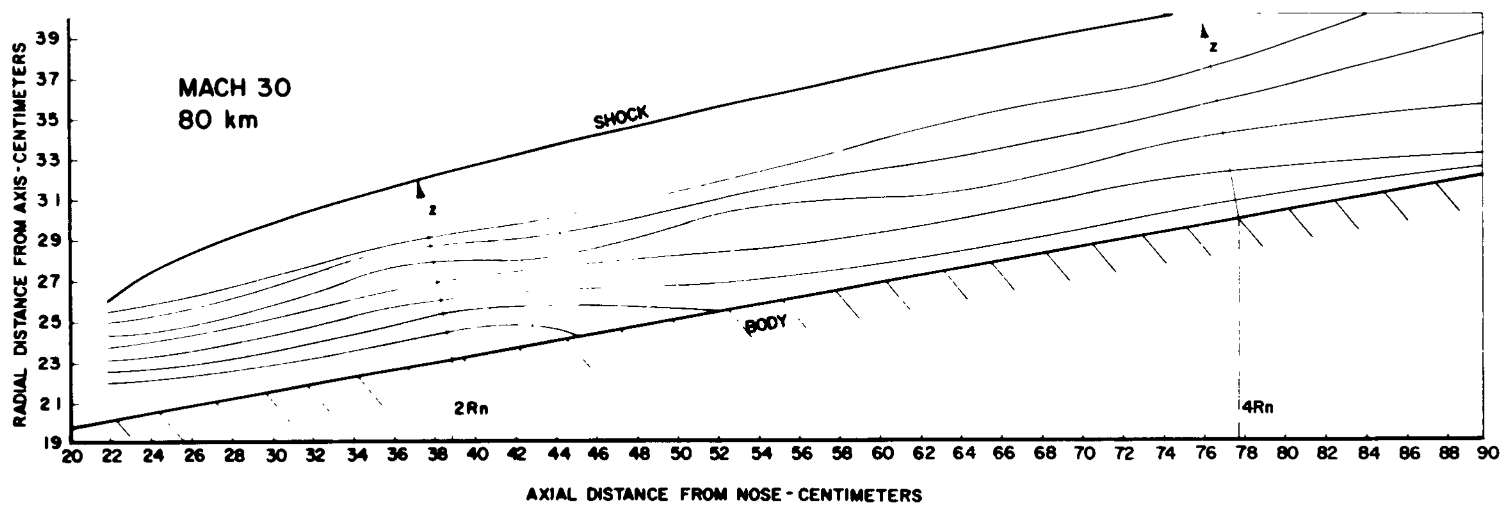

FIGURE 5. CONTOURS OF CONSTANT ϵ/ϵ_0

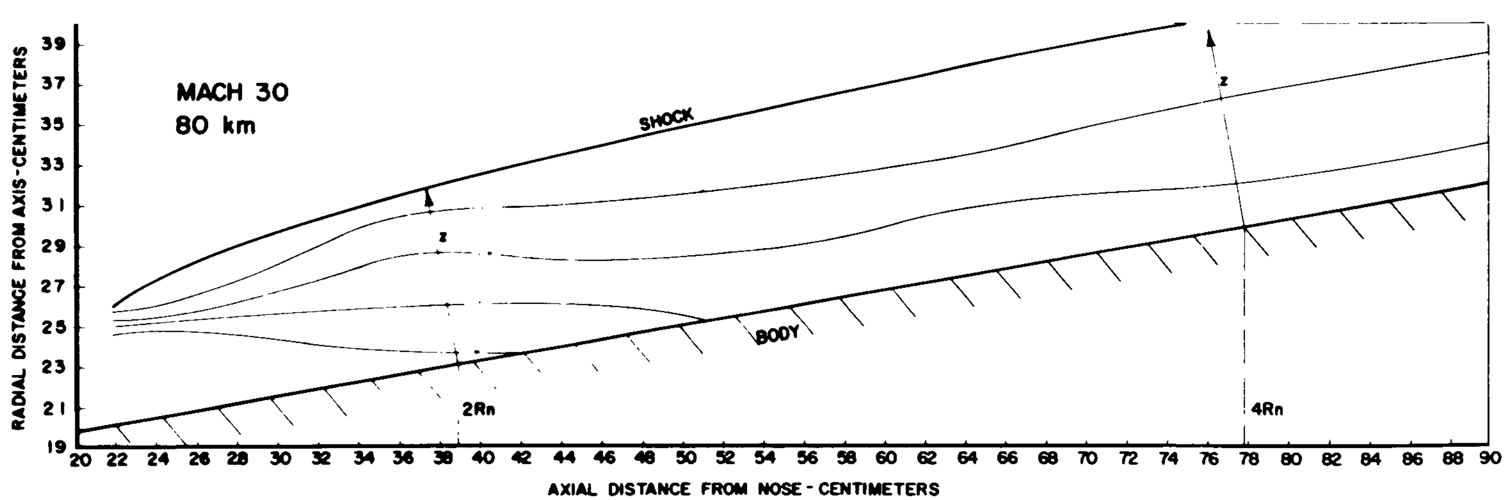

FIGURE 6. CONTOURS OF CONSTANT $\sigma/\epsilon_0\omega$

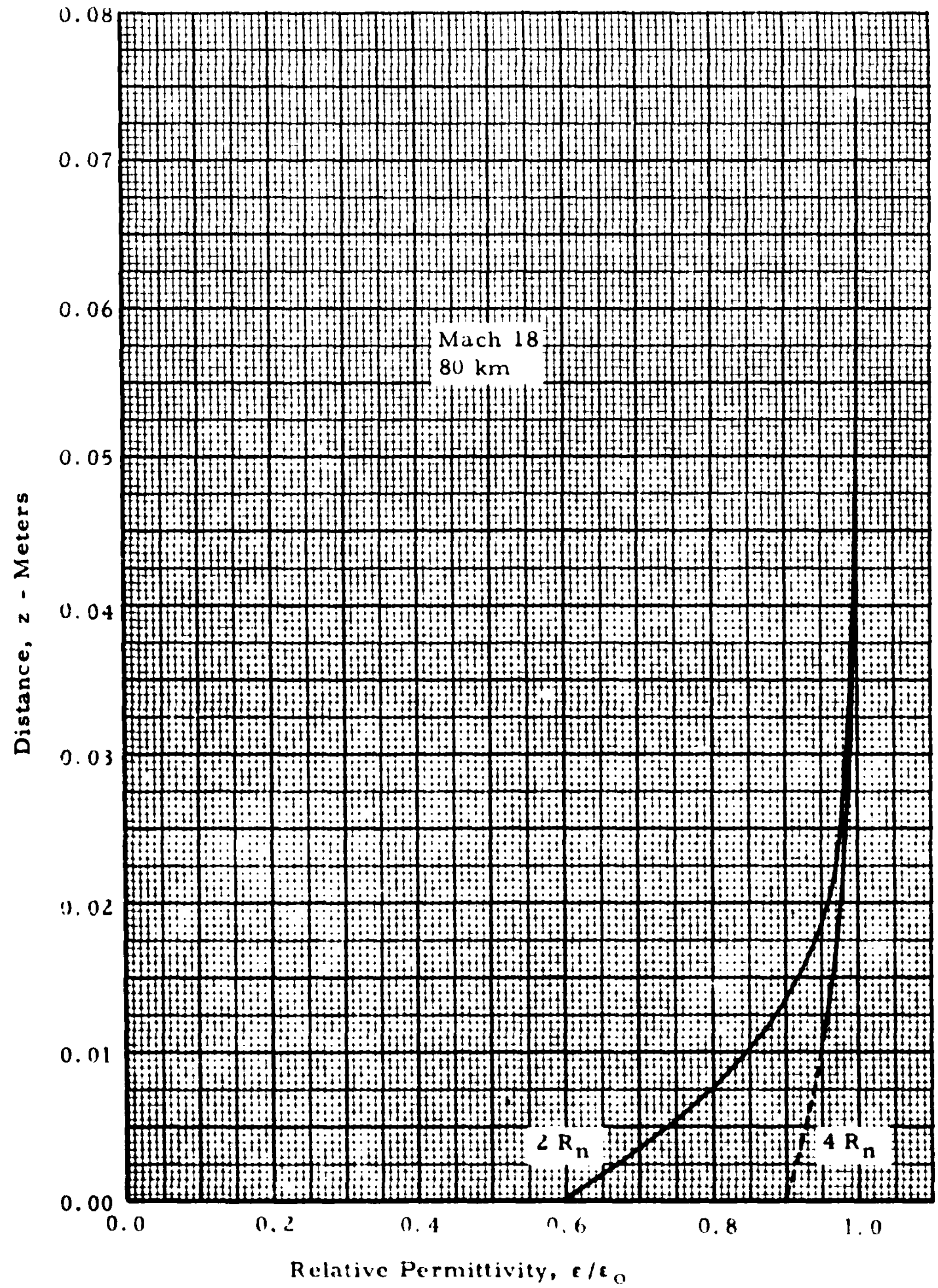

Figure 7

Permittivity Profiles at 2 R_n and 4 R_n

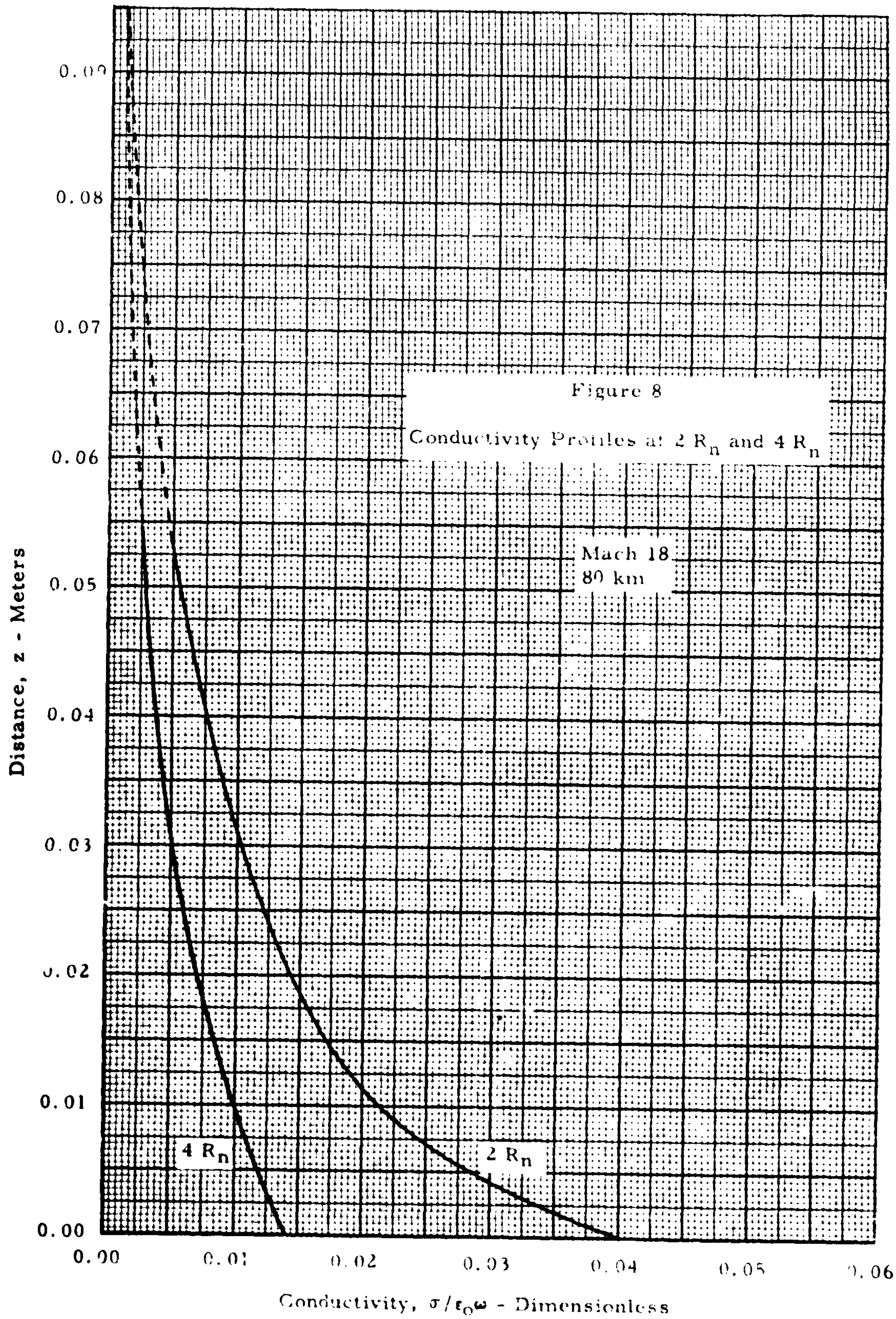
Figure 8
Conductivity Profiles at 2 R_n and 4 R_n
Mach 18
80 km
4 R_n
2 R_n
Distance, z - Meters
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.00
0.01
0.02
0.03
0.04
0.05
0.06
Conductivity, $\sigma/\epsilon_0\omega$ - Dimensionless

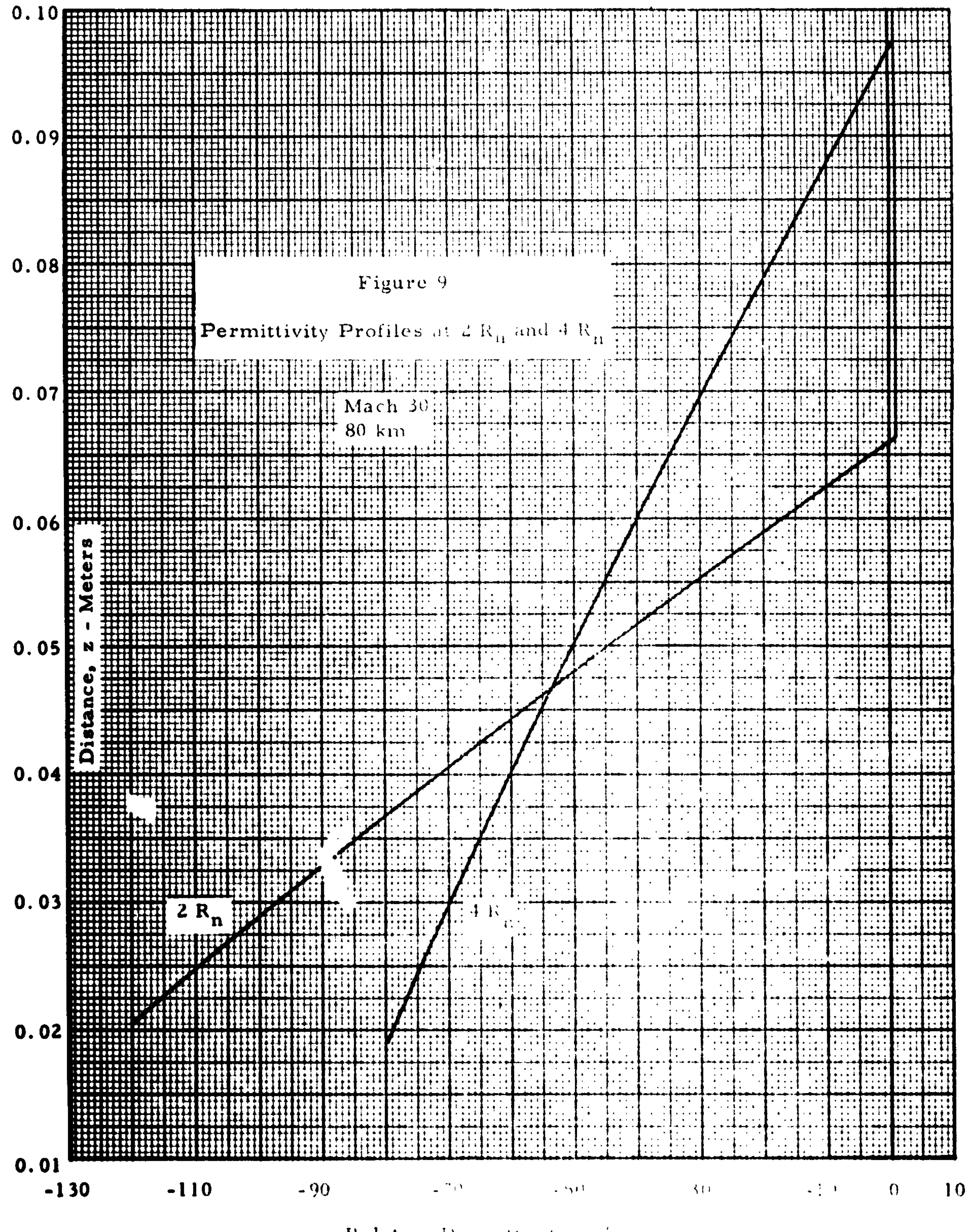

Figure 9

Permittivity Profiles at $2\ R_n$ and $4\ R_n$

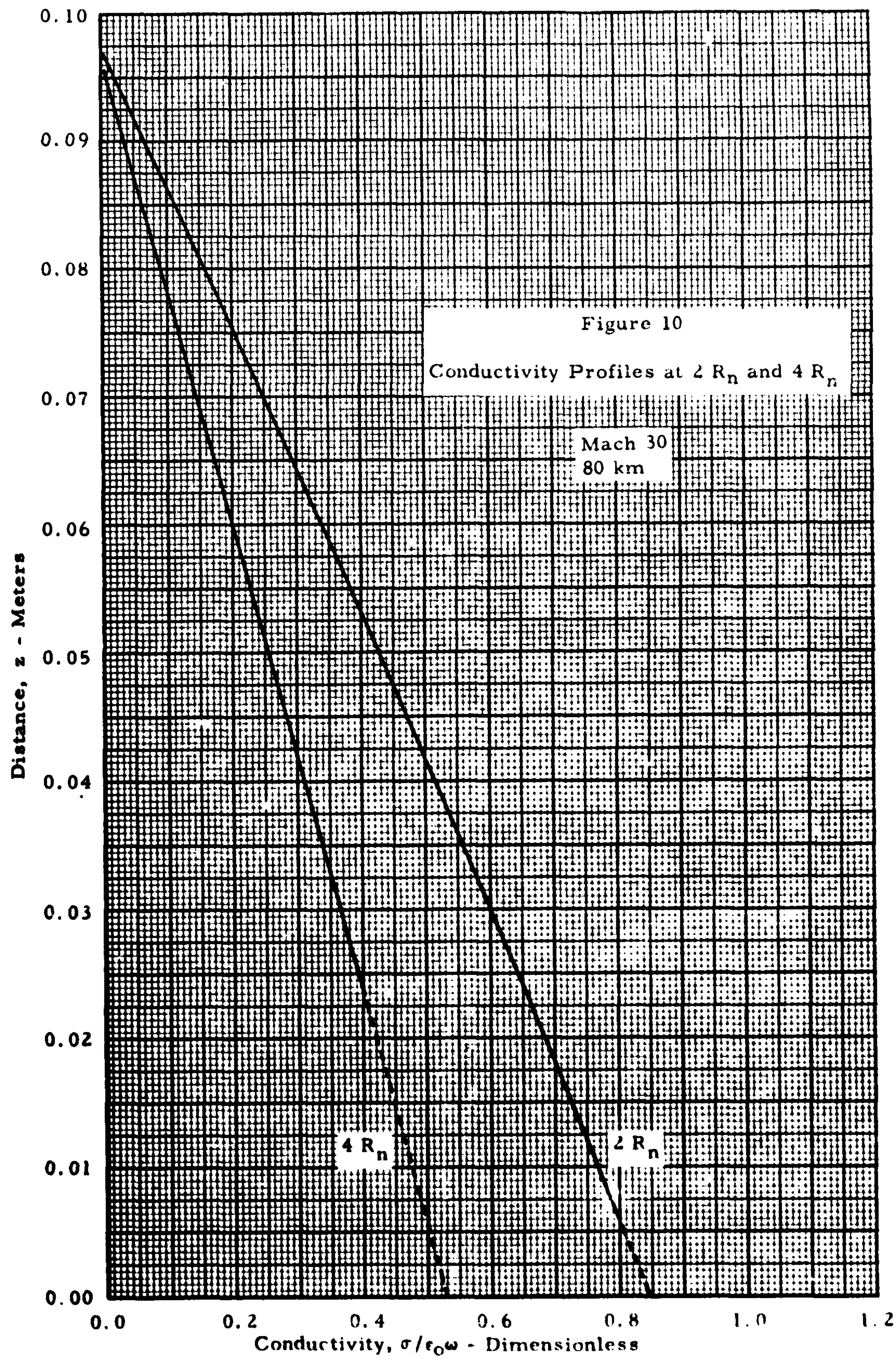
Figure 10
Conductivity Profiles at 2 R_n and 4 R_n
Mach 30
80 km
4 R_n
2 R_n
Distance, z - Meters
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Conductivity, $\sigma/\epsilon_0\omega$ - Dimensionless

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